

# Quantum Superinductor with Tunable Non-Linearity

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We report on the realization of a superinductor, a dissipationless element whose microwave impedance greatly exceeds the resistance quantum  $R_Q$ . The design of the superinductor, implemented as a ladder of nanoscale Josephson junctions, enables tuning of the inductance and its nonlinearity by a weak magnetic field. The Rabi decay time of the superinductor-based qubit exceeds  $1\mu\text{s}$ . The high kinetic inductance and strong nonlinearity offer new types of functionality, including the development of qubits protected from both flux and charge noises, fault tolerant quantum computing, and high-impedance isolation for electrical current standards based on Bloch oscillations.

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Superinductors are desired for the implementation of the electrical current standards based on Bloch oscillations [1, 2], protection of Josephson qubits from the charge noise [3, 4], and fault tolerant quantum computation [5, 6]. The realization of superinductors poses a challenge. Indeed, the “geometrical” inductance of a wire loop is accompanied by a sizable parasitic capacitance, and the loop impedance  $Z$  does not exceed  $\alpha R_Q$  [7], where  $\alpha = 1/137$  is the fine structure constant and  $R_Q = h/4e^2$  is the resistance quantum. This limitation does not apply to superconducting circuits whose kinetic inductance  $L_K$  is associated with the inertia of the Cooper pair condensate [8].

The kinetic inductance of a Josephson junction,  $L_K = (\Phi_0/2\pi)^2/E_J$ , scales inversely with its Josephson energy  $E_J$  [8] ( $\Phi_0 = h/2e$  is the flux quantum). The kinetic inductance can be increased by reducing the in-plane junction dimensions and, thus,  $E_J$ . However, this resource is limited: with shrinking the junction size, the charging energy  $E_C = e^2/2C$  ( $C$  is the junction capacitance) increases and the phase-slip rate  $\propto \exp[-(8E_J/E_C)^{1/2}]$  [8] grows exponentially, which leads to decoherence. Small Josephson energy (i.e., large kinetic inductance) can be realized in chains of dc SQUIDs frustrated by the magnetic field [9, 10]. However, the phase-slip rate increases greatly with frustration, and the chains do not provide good isolation from the environment. For the linear chains of Josephson junctions with  $E_J/E_C \gg 1$ , relatively large values of  $L_K$  (up to 300 nH [3]) have been realized in the phase-slip-free regime; further increase of the impedance of these chains is hindered by the growth of their parasitic capacitance. Also, the linear chains, as well as the nanoscale superconducting wires with a kinetic inductance of  $\sim 10\text{ nH}/\mu\text{m}$  [11, 12], are essentially linear elements whose inductance is not readily tunable (unless large currents are applied).

We propose a novel superinductor design that has several interesting features. This circuit can be continuously tuned by a weak magnetic field between the regimes char-

acterized by a low linear inductance and a very large nonlinear inductance. Importantly, the large impedance  $Z \gg R_Q$  is realized when the decoherence processes associated with phase slips are fully suppressed. This combination of strong nonlinearity and low decoherence rate is an asset for the development of high-performance superconducting qubits and controllable coupling between qubits.

The studied circuit is a “ladder” of nanoscale Josephson junctions frustrated by the magnetic flux  $\Phi$  [Fig. 1(a)]. Each unit cell of the ladder represents an asymmetric dc-SQUID with a single “small” junction with the Josephson energy  $E_{JS}$  in one arm and three “large” Josephson junctions with the Josephson energy  $E_{JL}$  in the other arm (the in-plane dimensions of both types of tunnel junctions do not exceed  $0.3 \times 0.3\mu\text{m}^2$ ). The adjacent cells are coupled via one large junction; the Josephson energy of the system,  $E_J(\varphi)$ , remains an even function of the phase difference  $\varphi$  across the ladder at any value of the flux (the benefits of this symmetry are discussed below).

For the Josephson junctions in the ladder “backbone,” the Josephson energy is two orders of magnitude larger than the charging energy (the junction parameters are summarized in Table I). In this case quantum fluctuations of the phase across individual junctions are small, and the dependence  $E_J(\varphi)$  [see Fig. 1(b)] can be obtained from the classical computation that minimizes  $E_J$  with respect to the phases of superconducting islands at a fixed  $\varphi$ . At zero frustration ( $\Phi = 0$ ) the energy  $E_J(\varphi)$  is approximately parabolic over the relevant range of  $\varphi$ . With an increase of the magnetic field, the curvature of  $E_J(\varphi)$  near  $\varphi = 0$  decreases. Provided the ratio of the Josephson energies for large and small junctions,  $r = E_{JL}/E_{JS}$ , is not too large, the curvature  $\partial^2 E_J(\varphi)/\partial\varphi^2|_{\varphi=0}$  vanishes at some critical flux  $\Phi_c$ . At this frustration the potential is strongly anharmonic (approximately quartic) and the kinetic inductance, which is inversely proportional to the curvature of the potential

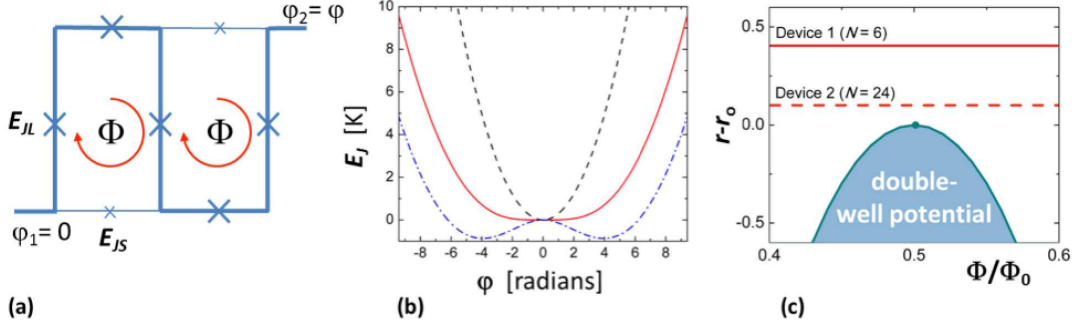


FIG. 1: (color online) Superinductor unit cells, potential energy, and phase diagram. (a) The unit cells of the tested device include “small” and “large” Josephson junctions with Josephson energies  $E_{JS}$  and  $E_{JL}$ , respectively. The “backbone” of the superinductor is shown as a bold line; the adjacent cells share large Josephson junctions. The cells are threaded by the same magnetic flux  $\Phi$ ; the phase difference across the device is  $\varphi_2 - \varphi_1 = \varphi$ . (b) The Josephson energy  $E_J(\varphi)$  of a six-cell ladder calculated within the quasiclassical approximation at  $\Phi = 0$  (dashed curve) and  $\Phi = \Phi_0/2$  (solid and dash-dotted curves);  $E_{JS} = 3.5$  K for all three curves,  $E_{JL} = 16.8$  K for dashed and solid curves and  $E_{JL} = 14$  K for the dash-dotted curve. For the optimal value of the ratio  $E_{JL}/E_{JS}$ , denoted as  $r_o$ , the dependence  $E_J(\varphi)$  becomes flat near  $\varphi = 0$  at  $\Phi = \Phi_0/2$  (solid curve). For smaller values of  $E_{JL}/E_{JS}$ , a double well potential is realized near full frustration. (c) “Phase diagram” of the ladders on the  $r - \Phi$  plane. The values of  $r_o$  are 4.1 and 4.5 for the ladders with  $N = 6$  and 24, respectively. The values of  $r - r_o$  for the studied devices are shown as horizontal lines.

$L_K \propto [\partial^2 E_J(\varphi)/\partial \varphi^2]^{-1}$  [8] in its minimum, diverges.

The optimal regime of operation for the studied superinductor and superinductor-based qubits is realized when the curvature  $\partial^2 E_J(\varphi)/\partial \varphi^2|_{\varphi=0}$  vanishes exactly at full frustration ( $\Phi = \Phi_0/2$ ). Indeed, because the energy is an even function of the flux at full frustration, the device in this regime is insensitive in the first order to the flux noise [13]. This allows for simultaneous realization of the maximum inductance (i.e., the maximum fluctuations of the phase across the ladder) and the minimal coupling to the flux noise. This regime corresponds to the optimal value of the ratio  $E_{JL}/E_{JS}$ , which we denote as  $r_o$ . The  $r_o$  value, calculated within the classical approximation ( $E_C = 0$ ), varies between 3 for a single unit cell and 5 for a very long ladder. Quantum fluctuations renormalize  $E_J(\varphi)$  and reduce  $r_o$  by  $\delta r$ ; for our devices  $\delta r \sim 0.5$ -0.7.

We have experimentally studied two types of ladders with the number of unit cells  $N = 6$  and 24. Short ladders with  $N = 6$  allow for direct comparison between experimental data and simulations based on the numerical diagonalization of the system Hamiltonian. Because these simulations are infeasible for a larger number of unit cells, the ladders with  $N = 24$  were treated within the quasiclassical approximation. These longer ladders demonstrate the potential of our novel approach: their microwave impedance exceeds  $R_Q$  by an order of magnitude.

The ladders and the readout circuits were fabricated using multiangle electron-beam deposition of Al films through a liftoff mask (see Supplemental Material IA for details). The in-plane dimensions of small and large junctions were  $0.16 \times 0.16 \mu\text{m}^2$  and  $0.30 \times 0.30 \mu\text{m}^2$ , respec-

tively. The unit cell size was  $3 \times 5 \mu\text{m}^2$ , and the flux  $\Phi = \Phi_0/2$  was realized in the magnetic field  $B \approx 0.7$  G. The ladders parameters are listed in Table I. Several devices with systematically varied values of  $r$  were fabricated on the same chip and inductively coupled to the same microstrip line. The devices could be individually addressed due to their different resonance frequencies.

The effective kinetic inductance of the ladder,  $L_K = (\omega_{01}^2 C_K)^{-1}$ , was calculated from the measured frequency  $\omega_{01}$  of the  $|0\rangle \leftrightarrow |1\rangle$  transition in the resonance

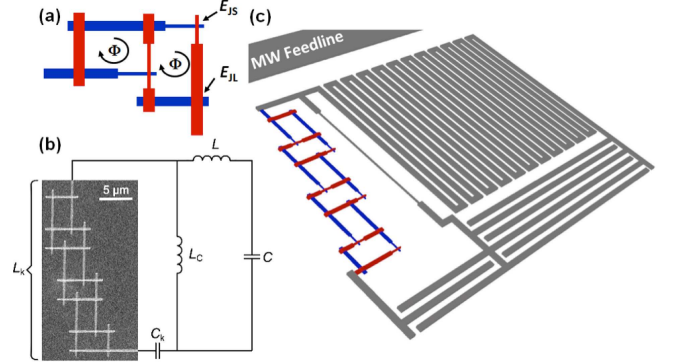


FIG. 2: (color online) Schematic description of the on-chip circuit. (a) Topology of two unit cells of the ladder. Josephson contacts are formed at the intersections of the bottom (horizontal) and top (vertical) electrodes. (b) The circuit diagram of the superinductor and  $LC$  resonators coupled via the kinetic inductance  $L_K$  of a narrow superconducting wire. The micrograph shows a ladder with six unit cells. (c) The on-chip circuit layout of two resonators inductively coupled to the microwave (MW) feedline.

TABLE I: Josephson junction parameters.  $E_{CS}$  and  $E_{CL}$  are the charging energies for the small and large junctions.

Device	Number of unit cells	$E_{JS}$ , K	$E_{CS}$ , K	$E_{JL}$ , K	$E_{CL}$ , K	$r =$ $E_{JL}/E_{JS}$	$C_K$ , fF	$L_C$ , nH	$L_K(\Phi = 0)$ , nH	$L_K(\Phi = \Phi_0/2)$ , nH
1	6	3.2	0.46	14.5	0.15	4.5	18	0.4	3.7	130
2	24	3.15	0.46	14.5	0.15	4.6	5	0.8	16	3 000

circuit formed by the ladder and the interdigitated capacitor. The capacitance  $C_K$  is larger than the capacitance of the interdigitated capacitor due to the parasitic capacitance to the ground. The resonance frequency of this circuit, referred below as the superinductor resonator, varies with the magnetic field by an order of magnitude (see below), whereas the bandwidth of the cryogenic preamplifier and cold circulators in our measuring setup is limited to the range of 3-10 GHz (the microwave setup is described in Supplemental Material IIB). To overcome this limitation, the superinductor resonator was coupled to a linear lumped-element  $LC$  resonator with a resonance frequency  $\omega_{LC}/2\pi \sim 7$  GHz via the coupling inductor  $L_C$  [a narrow superconducting wire, Figs. 2(b) and 2(c)]. The  $LC$  resonator is formed by an inductor (meandered  $2\text{-}\mu\text{m}$ -wide Al wire) with  $L = 5$  nH and an interdigitated capacitor ( $2\text{-}\mu\text{m}$ -wide fingers with  $2\text{-}\mu\text{m}$  spacing between them) with  $C = 100$  fF [Fig. 2(c)]. Both the superinductor and  $LC$  resonators are inductively coupled to a 2-port Al microstrip feedline with a  $50\text{-}\Omega$  wave impedance. When the superinductor resonator is excited by a second-tone microwave frequency  $\omega_2$ , its impedance changes due to nonlinearity; this results in a shift of the resonance of the  $LC$  resonator probed by the first-tone microwaves with  $\omega_1 \approx \omega_{LC}$ . The maximum shift occurs at the frequency  $\omega_2 = \omega_{01}$  that corresponds to the  $|0\rangle \leftrightarrow |1\rangle$  transition.

Figure 3 shows the resonance modes corresponding to the transition  $|0\rangle \leftrightarrow |1\rangle$  in the superinductor resonators with the 6-cell and 24-cell ladders. The insets in Fig. 3 show the avoided crossing between the lowest modes of the superinductor and  $LC$  resonators observed in the first-tone measurements (these avoided crossings illustrate the strength of coupling between these resonators). The measured values of the lowest-mode frequency  $\omega_{01}$  for the 6-cell ladder are in excellent agreement with simulations based on the numerical diagonalization of the circuit Hamiltonian (see Supplemental Material IIB). The only fitting parameter in these calculations was the ratio  $r = E_{JL}/E_{JS}$ , which is within 15% of the designed value. The nominal junction parameters for the 24-cell ladder also agree with the quasiclassical simulation of the dependence  $\omega_{01}(\Phi)$ .

Superinductor applications require that the frequency  $\omega_{01}$ , which corresponds to the “global” mode of the superinductor, should be much smaller than the frequency of its internal excitations,  $\omega_{\text{int}}$ . Far away from full frustration, the internal modes correspond to very high

frequencies of an order of the Josephson plasma frequency ( $\sim 100$  GHz for our samples). At full frustration,  $\omega_{\text{int}}$  decreases, but, according to our estimate, remains above 10 GHz for the 24-cell device.

The inductance at full frustration increases as  $r$  approaching its optimal value  $r_o$ , which depends on the number of unit cells (see Supplemental Material IIB):

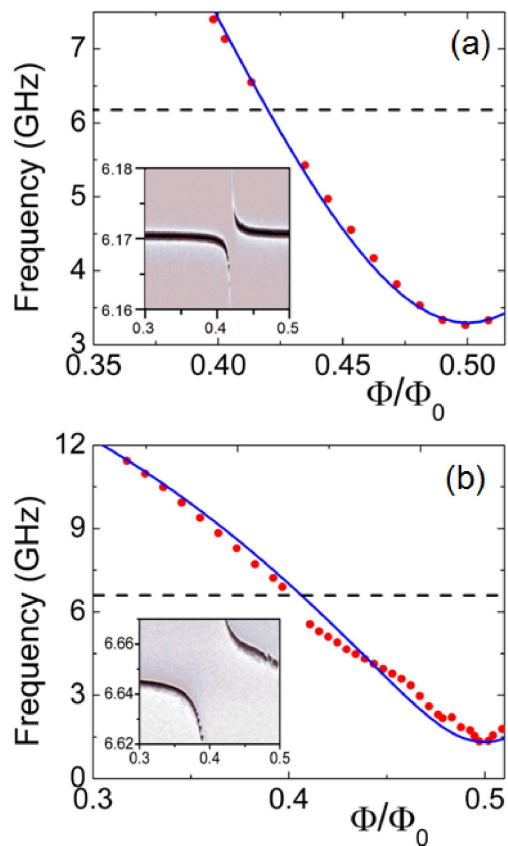


FIG. 3: (color online) Spectroscopic data for the ladders with  $N = 6$  (a) and  $N = 24$  (b). The resonance frequencies  $\omega_{01}(\Phi)/2\pi$  of the  $|0\rangle \leftrightarrow |1\rangle$  transition measured in the second-tone experiments are shown by red dots. The horizontal dashed lines correspond to the resonance frequency of the  $LC$  resonators. The blue curves represent the fits based on numerical diagonalization of the circuit Hamiltonian for device 1 (a) and the quasiclassical modeling for device 2 (a); for the simulation parameters, see Table I. The gray scale insets show the microwave amplitude  $|S_{21}|$  versus the first-tone microwave frequency  $\omega_1/2\pi$  and the normalized flux  $\Phi/\Phi_0$  measured near the avoided level crossings.

$r_o = 4.1$  for  $N = 6$  and  $r_o = 4.5$  for  $N = 24$ . Even though both devices 1 and 2 have similar values of  $r$  (i.e. nominally identical junction parameters), their proximity to the critical point is significantly different. For device 1 with  $r - r_o \approx 0.4$ , the inductance at full frustration exceeds that at zero field by a factor of 35. For device 2 with  $r - r_o \approx 0.1$ , this increase exceeds two orders of magnitude, and the inductance at full frustration is  $3 \mu\text{H}$  (for comparison, this is the inductance of a 3-meter-long wire). The total capacitance of the superinductor resonator,  $C_K = 5 \text{ fF}$  (see Table I), includes the capacitance of the interdigitated capacitor (2 fF) and the parasitic capacitance of all wires and the superinductor to the ground (3 fF) obtained by circuit modeling. By “meandering” the ladder and shrinking the tested moderately-sized ( $3 \times 5 \mu\text{m}^2$ ) unit cell, the parasitic capacitance can be reduced down to  $\sim 1 \text{ fF}$ ; the impedance of such a ladder approaches  $50 \text{ k}\Omega$  at  $\omega/2\pi = 3 \text{ GHz}$ .

The nonlinearity of the superinductor-based qubit increases dramatically with approaching the optimal working regime ( $r = r_o$  and  $\Phi = \Phi_0/2$ ). According to our quantum simulations, the nonlinearity factor  $\gamma = (\omega_{12} - \omega_{01})/\omega_{01}$  ( $\omega_{12}$  is the frequency of the  $|1\rangle \leftrightarrow |2\rangle$  transition) for the 24-cell ladder (device 2) with  $r - r_o \approx 0.1$  approaches 40 % at full frustration. Strong, tunable quartic nonlinearity of the studied superinductor is an asset for the qubit design. In particular, strong anharmonicity enables fast qubit operations and qubit readout due to a large energy difference between the  $|0\rangle \leftrightarrow |1\rangle$  and  $|1\rangle \leftrightarrow |2\rangle$  transitions [14].

In order to demonstrate the high quality of our superinductor, we have measured Rabi oscillations in the qubit formed by the superinductor and the capacitor  $C_K$  [Fig. 4(a)]. In these measurements, we have monitored the phase shift of the  $LC$  resonance while the superinductor resonator was excited by pulsed microwaves with  $\omega_2 = \omega_{01}$ . The quantum nature of these oscillations was verified by observing the linear dependence of the Rabi frequency on the amplitude of the microwave field [Fig. 4(b)]. The observed decay time of Rabi oscillations exceeded  $1 \mu\text{s}$  and was limited by the energy relaxation time [cf. Fig. 4(c)]. The dominant source of energy relaxation is the intentional inductive coupling to the  $LC$  resonator and the microwave feedline.

The intrinsic decoherence rate of the qubit is expected to be very low. Because the curvature  $\partial^2 E_J(\varphi)/\partial\varphi^2$  (which controls the position of energy levels) has a minimum at full frustration, one expects that the flux noise does not affect the qubit in the linear order. Another common source of dephasing in a chain of superconducting islands coupled by Josephson junctions is the phase-slip processes in combination with ubiquitous fluctuations of offset charges on the islands [15]. Because of the Aharonov-Casher effect [3, 16–18], these two factors result in the decoherence which is directly proportional to the phase-slip rate. In the studied devices this rate, being

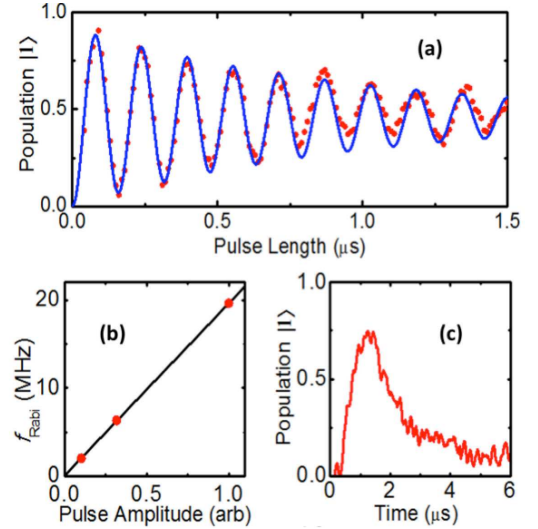


FIG. 4: (color online) (a) Rabi oscillations of the population of the first excited level of the superinductor resonator in device 1. The phase shift of the  $LC$  resonance was measured while the superinductor resonator was excited by the second-tone pulsed microwaves with  $\omega_2 = \omega_{01}$ . The data are shown for the phase  $\varphi = 2\pi\Phi/\Phi_0 = 0.94\pi$ . The solid line represents the fit with the Rabi decay time  $1.4 \mu\text{s}$ . (b) Dependence of the Rabi frequency on the amplitude of microwaves with  $\omega_2 = \omega_{01}$ . (c) The response of the  $LC$  resonator (measured at  $\omega_1 = \omega_{LC}$ ) to the excitation of the superinductor resonator by a  $0.4 \mu\text{s}$  second-tone pulse.

proportional to  $\exp[-c(E_{JL}/E_{CL})^{1/2}]$ , where  $c \approx 2.5\text{--}2.8$ , is expected to be negligible due to a large ratio  $E_{JL}/E_{CL}$  ( $\approx 100$ ) for the junctions that form the ladder backbone [Fig. 1(a)]. Recent work [19] demonstrated that linear chains of Josephson junctions with  $E_J/E_C \approx 100$  are phase-slip-free and exhibit inductances up to  $0.3 \mu\text{H}$ .

We envision many applications for the designed superinductor: this element has the potential to reduce the charge noise sensitivity of Josephson qubits, enable implementation of the fault tolerant qubits, and provide sufficient isolation for the electrical current standards based on Bloch oscillations. The ability to transform this element from an inductor with an almost linear response into a very nonlinear superinductor by tuning the magnetic field can facilitate controllable coupling between qubits. In a moderately or strongly nonlinear regimes, the superinductor-based resonator can also operate as a qubit with a low decoherence rate. Being combined with a small Josephson junction, the superinductor can be used as an adiabatic switch — an element whose impedance changes exponentially with magnetic field [5, 20], which is crucial for the fault tolerant qubit operations [6].

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## Appendix A: Experimental details

### 1. Device fabrication

The superinductor, the lumped-element  $LC$  resonator, and the microstrip feedline line were fabricated within the same vacuum cycle using multi-angle electron-beam deposition of Al films through a nanoscale lift-off mask. To minimize the spread of the junction parameters, we have used the so-called “Manhattan-pattern” bi-layer lift-off mask formed by a 400-nm-thick e-beam resist (the top layer) and 50-nm-thick copolymer (the bottom layer).

In this technique, the circuit is formed by aluminum strips of a well-controlled width intersecting at right angles (thus, the “streets” and “avenues”). After depositing the photoresist on an undoped Si substrate and exposing the pattern with e-beam lithography, the sample was placed in an ozone asher to remove any traces of the photoresist residue. This step is crucial for reducing the spread in junction parameters.

The substrate was then placed in an oil-free high-vacuum chamber with a base pressure of  $5 \times 10^{-9}$  mbar. The rotatable substrate holder is positioned at an angle of  $45^\circ$  with respect to the direction of e-gun deposition of Al. During the first Al deposition, the substrate was positioned such that the “streets” were orthogonal to the deposition direction. If the width of lines in the e-beam resist forming the “streets” is less than the mask thickness ( $0.45 \mu\text{m}$ ), no metal is deposited in the “streets” during this deposition, whereas the “avenues” are fully covered with Al. Alternatively, if the lines forming the “streets” are wider than  $0.5 \mu\text{m}$ , both the “streets” and “avenues” are covered with metal in the first deposition. The lines narrower than  $0.5 \mu\text{m}$  were used to form Josephson junctions, the wider lines were used for the fabrication of meander-shaped inductors. Without breaking vacuum, the surface of the bottom Al layer with a thickness of 20 nm was oxidized at  $\sim 100$  mTorr of dry oxygen for 5 minutes.

After evacuating oxygen, the substrate holder was rotated by  $90^\circ$ , and the second 60-nm-thick Al film was deposited; this layer forms top electrodes of Josephson junctions, and increases the total thickness of the meander-shaped inductors and interdigitated capacitors. Finally, the sample was removed from the vacuum chamber and the lift-off mask was dissolved in the resist remover. The spread of the resistances for the nominally identical JJs with an area of  $0.15 \times 0.15 \mu\text{m}^2$  did not exceed 10 %.

### 2. Measurement technique

The microwave response of the coupled superinductor and  $LC$  resonators was probed by measuring both the phase and amplitude of the microwaves traveling along a microstrip feedline coupled to the resonators. A simplified schematic of the microwave circuit is shown in Fig. 5. The cold attenuators and low-pass filters in the input microwave line prevented leakage of thermal radiation into the resonator. On the output line, two cryogenic Pamtech isolators ( $\sim 18$  dB isolation between 3 and 12 GHz) anchored to the mixing chamber were used to attenuate the  $\sim 5$  K noise from the cryogenic amplifier.

The probe microwaves at frequency  $\omega_1$ , generated by a microwave synthesizer (Anritsu MG3694B), were coupled to the cryostat input line through a 16 dB coupler. These microwaves, after passing the sample, were amplified by a cryogenic HEMT amplifier (Caltech CITCRYO

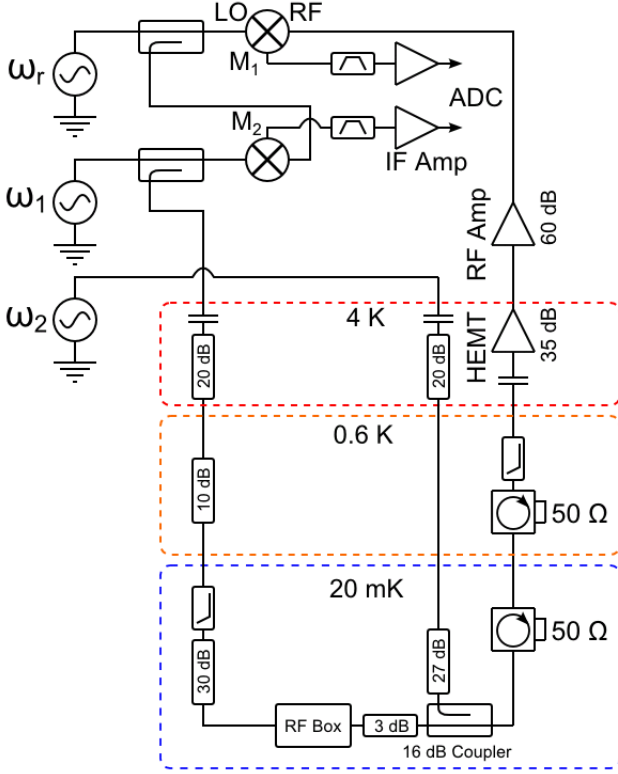


FIG. 5: Simplified circuit diagram of the measurement setup. The microwaves at the probe frequency  $\omega_1$  are transmitted through the microstrip line coupled to the superinductor and  $LC$  resonators. The microwaves at  $\omega_r$ , after mixing with the microwaves at  $\omega_1$ , provided the reference phase  $\phi_0$ . The signal at  $\omega_1$  is amplified, mixed down to an intermediate frequency  $\omega_1 - \omega_r$  by mixer  $M_2$ , and digitized by a fast digitizer (ADC). The second channel of the ADC is used to digitize the signal from mixer  $M_2$ . The microwaves at the second-tone frequency  $\omega_2$  were coupled to the transmission line via a 16 dB coupler.

1-12, 35 dB gain between 1 and 12 GHz) and two 30 dB room-temperature amplifiers. The amplified signal was mixed by mixer  $M_1$  with the local oscillator signal at frequency  $\omega_r$ , generated by another synthesizer (Gigatronics 910). The intermediate-frequency signal  $A(t) = A \sin(\Omega t + \phi_0) + A_N(t)$  (where  $A_N(t)$  is the noise term) at  $\Omega = (\omega_1 - \omega_r)/2\pi = 30$  MHz was digitized by a 1 GS/s digitizing card (AlazarTech ATS9870). The signal was digitally multiplied by  $\sin(\Omega t)$  and  $\cos(\Omega t)$  averaged over  $\sim 10^6$  periods, and its amplitude  $A$  (proportional to the microwave amplitude  $|S_{21}|$ ) and phase  $\phi$  were extracted as  $A = \langle [A(t) \sin(\Omega t)]^2 + [A(t) \cos(\Omega t)]^2 \rangle^{1/2}$  and  $\phi = \arctan\{ \langle [A(t) \sin(\Omega t)]^2 \rangle / \langle [A(t) \cos(\Omega t)]^2 \rangle \}$ , respectively. The reference phase  $\phi_0$  (which randomly changes when both  $\omega_1$  and  $\omega_2$  are varied in measurements) was found using similar processing of the low-noise signal provided by mixer  $M_2$  and digitized by the second channel of the ADC. This setup enables accurate measurements of small changes  $\phi - \phi_0$  unaffected by the phase jitter be-

tween the two synthesizers. The low noise of this setup allowed us to perform measurements at a microwave excitation level of  $-133$  dBm which corresponded to a sub-single-photon population of the tank circuit. In the second-tone measurements, the superinductor resonator was excited by the microwaves at frequency  $\omega_2$  propagating along the same microwave feedline that was used for the microwave transmission at  $\omega_1$ .

## Appendix B: Theoretical analysis

Here we explain in detail the nature of the large inductances of the Josephson ladders studied in this work, the role played by the field induced frustration and the numerical computations which results we compared with the data in the main text. We begin our discussion with the analysis of the classical (the Coulomb energy  $E_C = 0$ ) model. We then discuss the computation that takes into account quantum fluctuations induced by a non-zero  $E_C$ .

### 1. Classical analysis

The main idea of the superinductor design becomes more transparent in the case of a fully frustrated ( $\Phi = \Phi_0/2$ ) long ladder shown in Fig. 6(a). As we explain below, the fully frustrated regime is also very important experimentally because in this regime the quantum states of the ladder are less sensitive to the flux noise.

It is convenient to choose the gauge in which the magnetic field induces the phases  $2\pi\Phi/\Phi_0$  on smaller junctions. Because at full frustration these phases are equal to  $\pi$ , the shift of the ladder by one unit cell combined with its rotation about its axis transforms the ladder into itself, so the problem becomes translationally invariant. Furthermore, a large length of the ladder allows to neglect the boundary effects and focus on translationally invariant solutions.

Consider a small portion of the fully frustrated ladder shown in Fig. 6(a). The translational invariance implies that the solution is described by two phases across large junctions:  $\alpha$  at the “vertical” junctions and  $\beta$  at the “horizontal” ones. The total phase across the whole ladder is  $\varphi = N(\alpha + \beta)$ , where  $N$  is the number of rungs. The energy per unit cell that contains two large and one small junctions is given by

$$E(\alpha, \beta) = -E_{JL}(\cos \alpha + \cos \beta) - E_{JS} \cos(\pi - 2\alpha - \beta). \quad (\text{B1})$$

The dependence  $E(\alpha, \beta)$  is shown in Fig. 7(a). At  $E_{JL} \gg E_{JS}$  the first term in Eq. (B1) dominates and  $E(\alpha, \beta)$  has a minimum at  $(\alpha, \beta) = (0, 0)$ . The expansion near this

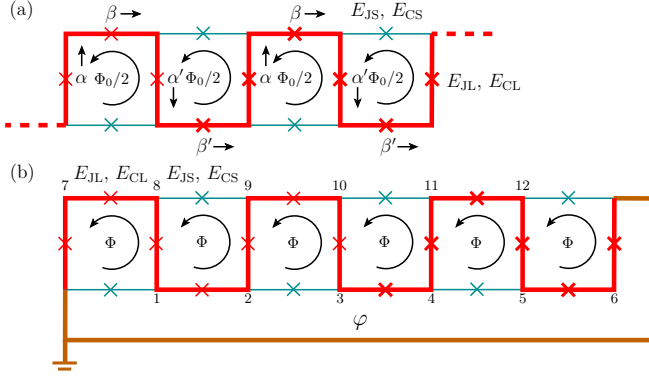


FIG. 6: (a) A small portion of a long fully frustrated Josephson ladder. Large junctions with the Josephson energy  $E_{JL}$  and the Coulomb energy  $E_{CL} \ll E_{JL}$  are shown in red thick lines; they form the ladder backbone. Small junctions with the Josephson energy  $E_{JS}$  and the Coulomb energy  $E_{CS} \ll E_{JS}$  are shown in cyan thin lines. The frustration of each cell is controlled by the dimensionless parameter  $\Phi/\Phi_0$ . (b) Schematic diagram of a six cell ladder (device 1). The ends of the ladder are grounded: the phase difference across the ladder is  $\varphi$ .

point gives

$$E^{(2)}(\alpha, \beta) = \frac{1}{2} \begin{bmatrix} \alpha & \beta \end{bmatrix} \begin{bmatrix} E_{JL} - 4E_{JS} & -2E_{JS} \\ -2E_{JS} & E_{JL} - E_{JS} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}, \quad (\text{B2})$$

which is a quadratic form with eigenvalues  $E_{JL}$  and  $E_{JL} - 5E_{JS}$ . The second eigenvalue changes sign as the ratio  $r = E_{JL}/E_{JS}$  decreases, signaling the instability and appearance of a nontrivial ground state. The critical value of the ratio,  $r_o = 5$ , corresponds to the optimal working point, as explained below. At this point the function  $E(\alpha, \beta)$  becomes flat along the eigenvector  $(2, 1)$ . We introduce new coordinates in the “flat” and “steep” directions:  $\gamma = (2\alpha + \beta)/\sqrt{5}$ ,  $\delta = (\alpha - 2\beta)/\sqrt{5}$ . Figure 7(b) shows the energy plotted along the flat direction. Neglecting the phase deviations in the steep direction, the total phase is related to  $\gamma$  by  $\varphi = 3N/\sqrt{5}\gamma$ .

At  $r > r_o$  the ground state values of phases  $\gamma$  and  $\varphi$ ,  $\gamma_0$ , and  $\varphi_0$ , are zero. Deviations of the total phase from  $\varphi_0 = 0$  result in the quadratic increase of the energy

$$E^{(2)}(\varphi) = \frac{5}{18N} E_{JS} (r - r_o) \varphi^2.$$

At the optimal point ( $r = r_o$ ) these deviations vanish, and the inductance, which is inversely proportional to  $d^2E/d\varphi^2$ , becomes infinite in the quadratic approximation and is defined by the non-vanishing quartic term

$$E^{(4)}(\varphi) = \frac{625 - 17r}{48600N^3} E_{JS} \varphi^4 \Big|_{r=r_o} = \frac{1}{90N^3} E_{JS} \varphi^4.$$

At smaller  $r < r_o$  the ground state corresponds to a non-zero value of  $|\gamma_0(r)| \propto \sqrt{r_o - r}$  that translates into a non-

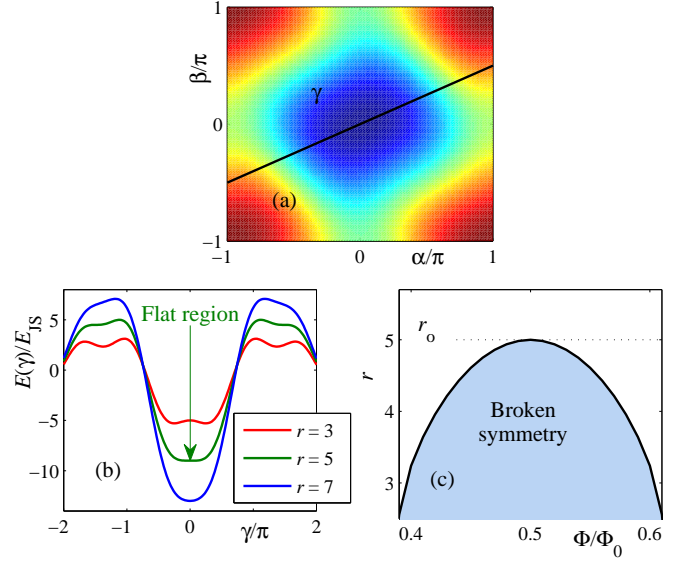


FIG. 7: (a) Energy per elementary cell  $E(\alpha, \beta)$  (B1) as a function of  $\alpha$  and  $\beta$  at the optimal point  $r = r_o$ . The black line corresponds to the flat direction parametrized by  $\gamma$ . (b) The dependence of energy in the flat direction for different values of  $r$ . (c) Phase diagram of the model in classical approximation in dimensionless variables  $r = E_{JL}/E_{JS}$  and  $\Phi/\Phi_0$ . Quantum fluctuations discussed in section B 2 shift the transition line down but affect neither the long range order nor the phase transition.

zero  $\varphi_0 = \pm 3N\gamma_0/\sqrt{5}$ . The appearance of a non-zero  $\gamma_0$  is equivalent to a phase transition.

The translational invariance discussed above is violated away from full frustration. At arbitrary frustration, the ladder remains symmetric under the translation by two unit cells. Denoting the phases across the large junctions in the first cell by  $\alpha, \beta$  and in the second cell by  $\alpha', \beta'$  we obtain the classical energy per two rungs

$$E(\Phi) = -E_{JL} [\cos \alpha + \cos \beta + \cos \alpha' + \cos \beta'] - E_{JS} \left[ \cos\left(\frac{2\pi\Phi}{\Phi_0} - \alpha_+ - \beta\right) + \cos\left(\frac{2\pi\Phi}{\Phi_0} + \alpha_+ + \beta'\right) \right], \quad (\text{B3})$$

where  $\alpha_+ = \alpha + \alpha'$ . This energy does not change under the combined transformation  $\alpha \rightarrow \alpha'$ ,  $\beta \rightarrow \beta'$ , and  $\Phi \rightarrow -\Phi$ , which correspond to the shift by one unit cell and simultaneous change in the sign of the magnetic field. The total phase difference along the ladder  $\varphi = N(\alpha_+ + \beta_+)/2$  is invariant under this transformation. Time inversion invariance implies that the ground state energy does not change under the simultaneous change in sign in both  $\Phi$  and  $\varphi$ , the symmetry under the combined transformation implies that the ground state energy is an even function of  $\varphi$  at any field. This conclusion is based only on the symmetry of the problem and thus remains valid when quantum fluctuations are taken into account (see section B 2).

The energy (B1) is minimal at  $\alpha_- = \alpha' - \alpha = 0$  and  $\beta_- = (\beta' - \beta)/2 \neq 0$  that has to be determined from the minimization of (B3). The minimum in  $\alpha_-, \beta_-$  is always steep, so fluctuations in this direction can be neglected. In this approximation the general expression for the energy (B3) is reduced to the form (B1) with the effective

$$E_{JS}^{(\text{eff})} = E_{JS} \cos(2\pi x + \beta_-),$$

where  $x = \Phi/\Phi_0 - 1/2$ .

At  $r > r_o$  the ratio  $E_{JL}/E_{JS}^{(\text{eff})}$  that controls the instability is large:  $E_{JL}/E_{JS}^{(\text{eff})} > r > r_o$ , so the ground state corresponds to zero phases. At  $r < r_o$  the instability occurs at non-zero  $x_c$ , defined by  $E_{JL}/E_{JS}^{(\text{eff})}(x_c) = r_o$  [see Fig. 7(c)]. At the point of instability we can determine the value of  $\beta_-$  assuming that  $\alpha_+ = \beta_+ = 0$ , we get

$$\beta_-(x) = \arctan \left[ \frac{\sin(2\pi x)}{r - \cos(2\pi x)} \right]. \quad (\text{B4})$$

The effective reduction of  $E_{JS}$  implies that at  $r > r_o$  the transition from the single minimum at  $\gamma = 0$  to double minimum at  $\pm\gamma_0$  occurs as a function of the field at  $x_c$  where

$$\cos[2\pi x_c + \beta_-(x_c)] = \frac{r}{r_o}. \quad (\text{B5})$$

The transition line defined by Eqs. (B4) and (B5) separates a symmetry-broken (ordered) phase from the phase with unbroken symmetry. We show the phase diagram obtained by the numerical solution of these equations in Fig. 7(c). The equation for the instability can be solved analytically for  $r_o - r \ll r_o$  (i.e.  $x \ll 1$ ):

$$x_c = \frac{r}{2\pi(r+1)} \arccos \left( \frac{r}{r_o} \right).$$

At the critical flux  $\Phi_c = (1/2 \pm x_c) \Phi_0$  the quadratic part of the energy vanishes which implies infinite inductance. Thus, the system can be tuned to the infinite inductance either by realizing  $r = r_o$  at full frustration, or by approaching  $\Phi_c$  at  $r < r_o$ .

In the absence of noises and disorder, these two methods of realization of the infinite inductance (and the phase transition) are equivalent. However, the real systems are subject to flux noise. Because the energy of the ground state is quadratic in flux deviations  $\delta\Phi$ , the effect of the flux noise on the system is minimized if the infinite inductance is realized at full frustration, or close to it. Thus, the optimal working point corresponds to the ratio  $E_{JL}/E_{JS} = r_o$ .

The finite length of the ladder brings in two effects associated with the boundaries. First, this may violate the symmetry which resulted in  $E(\varphi) = E(-\varphi)$  for all frustrations; this is crucial for the realization of infinite inductance at  $\Phi_c \neq \Phi_0/2$ . However, this effect is absent for the odd number of rungs (this was the reason

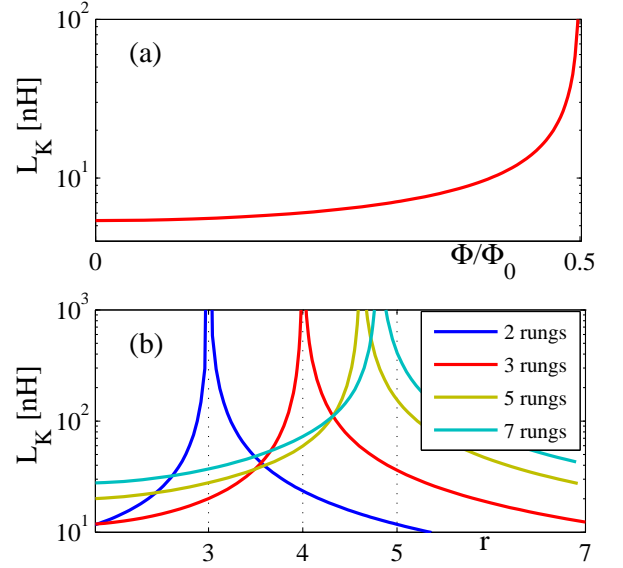


FIG. 8: (a) Inductance as a function of frustration (external magnetic field) in the classical approximation for a three-rung ladder at  $r = r_o$ . (b) The inductance as a function of the ratio  $r = E_{JL}/E_{JS}$  for ladder with 2, 3, 5, and 7 rungs. Optimal ratios  $r_o$ : 3 for 2 rungs, 4 for 3 rungs, 4.61 for 5 rungs, and 4.80 for 7 rungs.

for selecting the even number of unit cells in the studied ladders). In this case the ladder remains symmetric under the rotation by  $\pi$  about its center. The rotation does not change the magnetic fluxes but changes  $\varphi \rightarrow -\varphi$ , so in this case the function  $E(\varphi)$  remains even for all fluxes and  $r$ . Second, the finite-size effects decrease the value of the critical  $r$  at which the instability occurs at full frustration. To compute them for finite ladders we numerically computed the energy  $E(\varphi)$  for the ladders containing from 2 to 7 rungs. The dependences of the inductance  $L_K(\Phi, r) = (\partial^2 E / \partial \varphi^2)^{-1} \big|_{\varphi=\varphi_0}$  for different ladder lengths are shown in Fig. 8. The value of  $\varphi_0$  is defined by the minimum of the ground state energy  $E(\varphi)$  as a function of the phase  $\varphi$ . The inductance increases with frustration and diverges at full frustration  $\Phi = \Phi_0/2$  for  $r = r_o$ . For the ladders with six unit cells, the finite size effects lead to a small decrease of the optimal value to  $r_o = 4.8$ .

## 2. Quantum analysis

Non-zero (but small) charging energies result in quantum fluctuations of the phase of individual islands,  $\theta_i$ . These fluctuations have three effects. First, they might result in a phase slip in which the phase difference  $\eta_{ij} = \theta_i - \theta_j$  between two islands connected by a large Josephson junction changes by  $2\pi$  due to a quantum tunneling process. Second, they may result in renormalization of



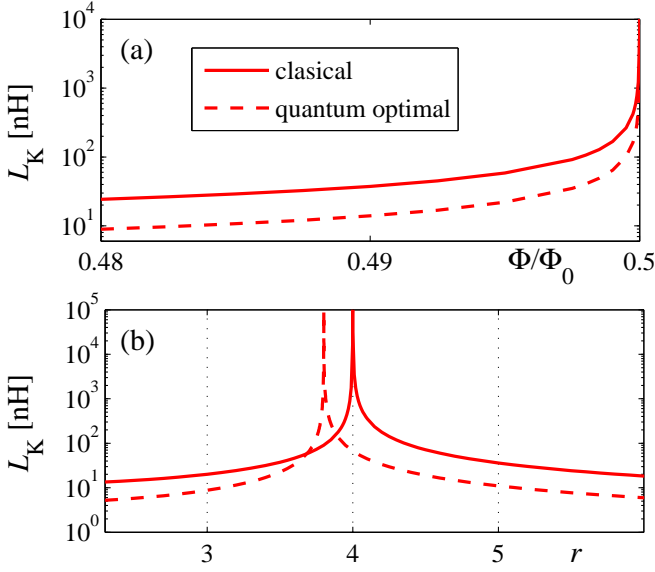


FIG. 9: Solid lines in panels (a) and (b) show the inductance computed in the classical approximation for a three-rung ladder as a function of frustration at  $r = r_o$  (a) and as a function of  $r$  for  $\Phi = \Phi_0/2$  (b). Dashed lines show the results of the full quantum computation for the same ladder. Quantum fluctuations shift position of the optimal point from its classical value ( $r_o = 4$ ) to  $r_o \approx 3.8$ . Dashed line in panel (a) shows the result of the full quantum computation at  $r = r_o$  which takes into account the shift of  $r_o$  due to quantum fluctuations.

the effective energy in the flat direction discussed in section B 1 and the shift of the phase boundary in Fig. 7(c). Finally, they may have a very large effect on the properties of long ladders close to the transition point at which critical fluctuations are expected to become important.

In the absence of frustration the amplitude of a phase slip across a single large junction,  $E_{ph} \approx 3.8 E_{CL}^{1/4} E_{JL}^{3/4} \exp(-\sqrt{8 E_{JL}/E_{CL}})$ , is exponentially small for the experimental values of  $E_{JL}/E_{CL} \approx 100$ . The frustration decreases the tunnel barrier resulting in larger phase slip amplitudes. However, this effect is numerically small. Namely, the dominant transitions correspond to the process in which the phase  $\beta$  changes between 0 and  $2\pi$  [see Fig. 7(a)], the action for this process is  $S \approx 2.5\sqrt{E_{JL}/E_{CL}}$ . Thus, the phase slip amplitudes remain negligible for the ladders with experimental parameters.

Small quantum fluctuations of the phase differences smear the classical potential. As a result the large inductance and the phase transition are realized at a smaller value of  $r_o$ . To estimate this effect we have diagonalized numerically the Hamiltonian describing small ladders. This diagonalization is convenient to perform in the charge basis in which the general Hamiltonian of the Josephson circuit acquires the form

$$\hat{H} = - \sum_{\langle i,j \rangle} \frac{J_{ij}}{2} (e^{-iA_{ij}} \mathbf{b}_i^+ \mathbf{b}_j^- + \text{H.c.}) + (2e)^2 \sum_{i,j} (C^{-1})_{ij} (n_i + q_i)(n_j + q_j), \quad (\text{B6})$$

where matrix  $J_{ij}$  describes Josephson couplings between islands and  $C_{ij}$  is the capacitance matrix of the system. For large junctions  $J_{i,i+1} = E_{JL}$ ,  $C_{i,i+1} = C_L$ , and  $E_{CL} = e^2/2C_L$ ; for small junctions  $J_{i,i+3} = E_{JS}$ ,  $C_{i,i+3} = C_S$ , and  $E_{CS} = e^2/2C_S$ . The operators  $\mathbf{b}_i^\pm$  increase (decrease) the number of Cooper pairs on each island:  $\mathbf{b}_i^\pm = |n_i \pm 1\rangle_i \langle n_i|$ . The junction matrix  $J_{ij}$  is symmetric and its diagonal terms are zeros. The capacitance matrix  $C_{ij}$  is positively defined with positive diagonal elements  $C_{ii}$  and negative off-diagonal elements  $C_{ij}$ ,  $i \neq j$ , so that its inverse  $(C^{-1})_{ij}$  is positively defined and its elements are all positive. The phases  $A_{ij}$  are induced by the magnetic field, the sum of  $A$ 's over a closed loop  $l$  of  $n$  Josephson junctions corresponds to the flux  $\Phi_l$  penetrating this loop.

For the numerical diagonalization we had to limit the number of charging states,  $n_q$  at each island. For the parameters similar to the experimental ones it is sufficient to keep  $n_q = 12$  to get the results with better than 1% accuracy. The effect of the quantum fluctuations on the inductance of a three rung chain with  $E_{JL}/E_{CL} = 40$  is shown in Fig. 9. We observe that quantum fluctuations reduce  $r_o$  by  $\delta r \approx 0.2$  down from its classical value, but the behavior of the inductance as a function of the frustration remains essentially the same. The importance of quantum fluctuations increases with the ladder length, so this shift becomes  $\delta r \approx 0.5$  for the ladders with six unit cells.

The inductance  $L$  can be calculated as the inverse curvature of the ground state energy  $E(\varphi)$ . Alternatively, it can be defined as  $L = 1/\omega_{01}^2 C$  for a superinductor shunted by a capacitor  $C$ . Here the frequency  $\omega_{01}$  is the excitation frequency of the superinductor resonator  $\omega_{01} = E_1 - E_0$ . In the limit of a large capacitance  $C$  both methods give the same result (for a linear circuit, the results of both methods are identical for any  $C$ ).

For the detailed comparison with the data for device 1 we have diagonalized the Hamiltonian of the ladder connected to a large capacitor. In this Hamiltonian we used the values of the charging energy of small junctions and their Josephson energy extracted from the independent measurements of the Cooper pair transistors containing nominally identical junctions with  $E_{CS} = 0.46$  K and  $E_{JS} = 3.2$  K. The value of the charging energy of the large junction that was calculated from the ratio of the areas of small and large junctions:  $E_{CL} = 0.15$  K. We used the designed value of the large capacitance  $C_K = 18$  fF. We then diagonalized the Hamiltonian of the ladders containing up to six rungs, determined

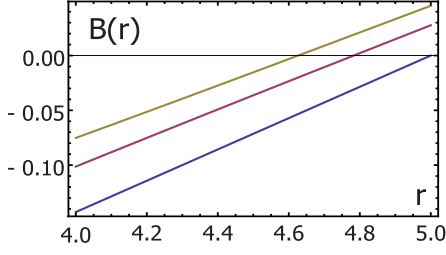


FIG. 10: Classical and quantum  $B(r)$ . From right to left: the results of classical computation and from the full diagonalization of Hamiltonians for three and four rungs ladders.

the excitation frequency of the superinductor resonator,  $\omega_{01}(\Phi) \equiv E_1 - E_0$ , and extrapolated the result to 7 rungs that cannot be computed directly due to an enormous size of the Hilbert space. For the experimental values of the charging energies  $r_o$  is shifted down to approximately 4.1. The 7-rung devices studied in this work (e.g. device 1) are characterized by the ratios  $r$  that are significantly above  $r_o$ , so the extrapolation to seven rungs works reasonably well. We found that in this parameter range the form of the dependence  $\omega_{01}(\Phi)$  in the interval  $0.4 < \Phi/\Phi_0 < 0.5$  is very similar for different  $r$ , whilst the absolute values of  $\omega_{01}(0.4\Phi_0)$  and  $\omega_{01}(0.5\Phi)$  change dramatically as a function of  $r$ . This dependence is plotted in Fig. 3(a). Fitting the ratio  $\omega_{01}(0.4\Phi_0)/\omega_{01}(0.5\Phi)$  to experimental values for device 1 gives for the ratio  $r = 4.5$  that is close to the designed value of 4.0.

The energy of the first excited states of longer device 2 was obtained in a two step procedure. In the first step we used exact diagonalization to find the energy functional of small quantum ladders (with 3-5 rungs) forming the loop penetrated by flux  $\Phi = \Phi_0/2$  and fit it to the general form expected in vicinity of the optimal point:

$$E(\varphi) = \frac{B_q(r)}{N} E_{JS} \varphi^2 + \frac{C_q(r)}{N^3} E_{JS} \varphi^4. \quad (\text{B7})$$

Here  $B_q(r)$  and  $C_q(r)$  are numerical coefficients that differ somewhat from their classical values,  $B_{cl}$  and  $C_{cl}$ . Comparison of the results of the computation with classical result shows that the main effect of quantum computation is a shift of the critical value of  $r$  (see Fig. 10),  $B_q(r) \approx B_{cl}(r + \delta r)$  with  $\delta r \approx 0.45$  and a modest renormalization of  $C_q(r) \approx 0.7 C_{cl}(r = r_o)$  that can be approximated by the constant in the relevant regime.

The procedure outlined above allowed us to determine the renormalization of the classical energy by short scale quantum fluctuations. In the second step we used Eq. (B7) to compute the potential energy of the long ladder of device 2. Solving finally the quantum problem of the phase fluctuations across the whole ladder characterized by the Hamiltonian

$$H = E(\varphi) + 4E_C q^2$$

we find the low energy excitations of the device. The results are shown in Fig. 3(b).